Review of Mathematical Statistics

Exercises 12.3, 12.4, 12.5, 12.6, 12.8, 12.11, 12.12, 12.13

1. Let $Y_1 < Y_2 < \cdots < Y_n$ be the order statistics of a random sample of size n from the uniform distribution $U(0;\theta)$. Consider Y_n as an estimator of θ.
   a. Show that $E(Y_n) = n\,\theta/(n+1)$ and $\text{var}(Y_n) = n\,\theta^2/\left((n+1)^2\,(n+2)\right)$.
   b. Find a constant c so that $T = cY_n$ is an unbiased estimator of θ.
   c. Compare the mean squared error of Y_n and T
2. Consider a normal population with known variance, σ^2. What is the probability that the length of a 95% confidence interval for μ assuming σ^2 known is less than the length of a similar interval assuming that σ^2 is unknown. Consider $n = 20$ and repeat the analysis using $n = 50$.
3. When choosing a product to purchase, what do you consider most: price or quality? In a poll of 2000 American adults conducted by Roper Starch Worldwide, 64% claim that they mainly base their buying decisions on price (*Tampa Tribune*, October 1993).
   a. Construct a 99% confidence interval for the true percentage of American adults who base their buying decisions more on price than on quality.
   b. How would the length of the interval change if confidence level was decreased to 95%?
4. Let $(X_1, X_2, \cdots, X_n))$ be a random sample from an exponential population with mean θ.
   a. Compare the asymptotic distributions of the sample mean and the sample median.
   b. Having observed a sample of size 100 where $\bar{x} = 20$, obtain a 95% confidence interval for θ when
5. A machine being used for packaging seedless raisins has been set so that on the average 15 ounces of raisins will be packaged per box. The quality control engineer wishes to test the machine settings and selects a sample of 30 consecutive raisin packages filled during the production process. Their weights are recorded below:

15.2	15.3	15.1	15.7	15.3	15.0	15.1	14.3	14.6	14.5
15.0	15.2	15.4	15.6	15.7	15.4	15.3	14.9	14.8	14.6
14.3	14.4	15.5	15.4	15.2	15.5	15.6	15.1	15.3	15.1

   Assume that the weight per box is normally distributed.

   a. Is there evidence that the mean weight per box is different from 15 ounces? (use $\alpha = 0.05$).

   b. Is there evidence that the standard deviation of weight per box is different from 0.25 ounces? (use $\alpha = 0.05$).

6. Stocks on the National Association of Security Dealers (NASD) system were analyzed in *Financial Analysts Journal* (Jan/Feb 1993). The annualized monthly returns (%) for a sample of 13 large-firm NASD stocks were computed and are summarized as follows: $\bar{x} = 13.5\%$, $s = 23.84$. Conduct a test of hypothesis to determine whether the mean annualized monthly return for large-firm NASD stocks exceeds 10%. Use $\alpha = 0.05$.
7. The management of the Tiger baseball team decided to sell only low alcohol beer in their ballpark to help combat rowdy fan conduct. They claimed that more than 40% of the fans would approve of this decision. Let p equal the proportion of Tiger fans on opening day that approved of this decision. Knowing that 550 fans out of a sample of 1278 said that they approved of this new policy what can you conclude?
8. A sample of 45 sales receipts from the university bookstore has $\bar{x} = 73.5$ and $s = 12.4$. Assume that sales receipts follow a normal distribution
   a. Use these values to perform a test of $H_0: \mu = 80$ against $H_1: \mu < 80$ with $\alpha = 0.05$. Calculate the p-value.
   b. Test $H_0: \mu = 80$ against $H_1: \mu \neq 80$ with $\alpha = 0.05$. Calculate the p-value. Why does the p-value change from the previous one.
   c. Define a 95% confidence interval for μ.
   d. Define a 95% confidence interval for σ.

Answers

1- a) … b) $c = (n+1)/n$ c) $mse_T(\theta) = \frac{1}{n(n+2)}\theta^2$; $mse_{Y_n}(\theta) = \frac{2}{(n+1)(n+2)}\theta^2$;

$mse_{Y_n}(\theta) > mse_T(\theta)$ for $n > 1$

2- n=20; p=0.5363; n=50; p=0.5222

3- a) (0.6124 ; 0.6676) or (0.6119; 0.6671) depending on the chosen method b) 0.7609 (first method) or 0.7614 (second method)

4- a) $\bar{X} \sim \gamma(n;\theta/n)$ or $\bar{X} \overset{\circ}{\sim} n(\theta;\theta/\sqrt{n})$; $M \overset{\circ}{\sim} n\left(\theta \ln 2;\theta/\sqrt{n}\right)$

b) (16.5953; 24.5809) or (16.7224; 24.8756) or (16.08; 23.92)

5- a) p-value=0.1369 do not reject H0 or $T_{obs} = 1.5298$, $c = \pm 2.045$

b) p-value=0.0000 reject H0 or $Q_{obs} = 76.395$, $q_1 = 16.047$, $q_2 = 45.722$

6- *p*-value=0.3031 do not reject H0 or $T_{obs} = 0.5293$, $c = 1.7823$ (one side test)

7- *p*-value=0.01336 Reject H0 and then more than 40% of the fans approve the decision

8- a) *p*-value=0.00051 b) *p*-value=0.001028 c) (69.7746; 77.2254) d) (10.2654; 15.6637)